Sampler of Bowl Problems, BAMM '95

Round 1

1

L Find the area of the region enclosed by the x-axis, the y-axis, and the line $y = x + 2$. **2**

R The circumference of a circle (measured in feet) is equal to its area (measured in square feet). Find the radius of the circle. **2**

2

L How many positive multiples of 11 less than 117 are there which are not multiples of 2, 3, 5, or 7? **1**

R Suppose a trapezoid has sides 5, 5, 5, and 11. Find its area. **32**

3

L There is a cube-shaped die in a cube-shaped hole (of the same size). You pick up the die, and then return it to the hole, but in the process, the die might be rotated. How many possible final positions are there for the die? **24**

R The sides of a triangle have lengths $\sqrt{5}$, $\sqrt{11}$, and 4. Find the area of the triangle. $\sqrt{55}/2$

4

L The number $2^{859433} - 1$, discovered in 1994, is the largest prime known today. Is the exponent 859433 also a prime? **yes**

R Jack walks around a circle of diameter 24 feet. Jill walks around a sqare of side 19 feet. Who has walked farther?

Jill

5

L At 12:30, what is the angle made by the minute hand and the hour hand of a standard clock?

165 degrees

R Five numbers are in geometric progression. If the third number is 13, what is the product of the first number with the fifth number?

169

6

L Let n =100,000,011 ("One hundred million and eleven"). What is the remainder when n^n is divided by 11?

1

R Find the area of a regular octagon which is circumscribed by a circle with radius 1.

$\sqrt{8} = 2\sqrt{2}$

Round 2

1

L In a particular election, everyone votes for either candidate A or candidate B. After 60% of the votes are counted, candidate A is leading 60% to 40%. What percentage of the remaining votes must B get in order to catch up to A?

65%

R Suppose there are 30-million Californians, and 5-million of them live in the Bay Area. Suppose also that 40% of all Californians have been to the East Coast, but 60% of all people in the Bay Area have been to the East

25%

Coast. If we select a random Californian who has been to the East Coast, then what is the probability that he or she is from the Bay Area?

2

L Suppose there is a seven-sided die with the property that when it is rolled, there is an equal probability of getting a 1,2,3,4,5,6, or 7. If this die is rolled twice, then what is the probability that the sum of the two rolls is even?

25/49

R Person 1 tells person 2 the lengths of two sides of a triangle. Person 2 then correctly concludes that the length of the third side can be any number between 2 and 10. What were the lengths of the first two sides?

6,4

3

L If one-and-a-half woodchucks can chuck one-and-a-third cords of wood in one-and-a-fourth hours, how many cords of wood can one woodchuck chuck in one hour?

32/45

R Suppose the base of a cone has circumference π , and suppose that the a line segment from the point of the cone to the circumference of the base has length 1. Find the surface area of the cone, not including the base.

$\pi/2$

4

L Two laser beams are aimed from the top of a vertical pole to the level ground below. The first beam makes an angle of 30 degrees with the ground. The second beam hits the ground 100 feet closer to the base of the pole, and makes a 60 degree angle with the ground. How tall is the pole?

$50\sqrt{3}$

R Suppose that two rays emanating from the point P form a 30 degree angle, and they are each tangent to a certain circle. The points of tangency divide the circle into two arcs. Find the degree-measure of each arc.

150 and 210 degrees

5

L Let s be the line segment in space connecting the point (1,0,0) to the point (-1,0,0). Let E be the set of points whose distance from s is not more than 1, but whose distance from the origin (0,0,0) is at least 1. Find the volume of E.

2π

R In an normally-shaped room, there is a light on the floor. If I hold a disc 4 feet above the light, then it casts a circular shadow on the ceiling of diameter 6 feet. If I then raise the disc 2 feet, what will be the new diameter of the shadow on the ceiling?

4

6

L *(use blackboard)* Find x, if

$$\log_2(\log_3(\log_4 x)) = 0.$$

64

R *(use blackboard)* Simplify the following expression.

$$\log_2(\log_2(\log_2(2^{4^8})))$$

4

Round 3

1

Which of the following triangles are obtuse? A 4,5,6 triangle or a $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$ triangle or a $4\sqrt{4}$, $5\sqrt{5}$, $6\sqrt{6}$ triangle?.

third one

2

B is a ball of radius $2r$ with a ball of radius r taken out of the inside. C is

$\frac{1}{\sqrt[3]{7}} = \frac{\sqrt[3]{49}}{7}$

a solid ball and has radius 1. Suppose that B and C have equal volumes. Find r.

3

Suppose $f(x) = 2x^5 + 7x^3 + x - 5$, and let $g(x)$ be the inverse of $f(x)$. Find a value of x for which $g(x) = 1$.

5

4

Suppose k is a positive integer that does not divide 1995. Let $f(x)$ denote the greatest integer less than or equal to x. For example, $f(\pi) = 3$. What is the maximum possible value of $kf(1995/k)$? (Read it "k times f of quantity 1995 over k".)

1994

5

Let n be the number whose base-7 representation is 30 digits, all 5's. What is the remainder when n is divided by 2?

zero

Round 4

1

If both diagonals of a convex quadrilateral have length 1, and the diagonals meet at a 45-degree angle, then what is the area of the quadrilateral?

$\sqrt{2}/4$

2

If x is a randomly selected real number between 0 and 1, y is a randomly selected real number between 0 and 1, what is the probability that their sum is less than or equal to 1?

1/2

3

In triangle ABC, $AB = 10$, $BC = 9$, and $\angle A = 60$ degrees. Find the sum of all possible lengths of AC.

10

4

Consider a standard clock with an hour-hand, minute-hand, and second-hand. Between 3:30 AM and 3:30 PM, how many times does the second-hand cross the minute-hand?

708

5

(use blackboard) What is the remainder when

$$x + 2x^2 + 3x^3 + 4x^4 + \cdots + 1995x^{1995}$$

is divided by $x + 1$?

-998

Round 5

1

You are given 6 positive integers, the largest of which is 6. The median is 3.5. What is the smallest possible value for the mean of these numbers?

19/6 = $3\frac{1}{6}$

2

What is the smallest positive integer n such that the integer 2^n, when written in ordinary base-10 notation, contains the digit 9?

12

3

If a regular octagon has side-length 1, find the distance between opposite sides. $\boxed{1+\sqrt{2}}$

4

Suppose x and y are real numbers such that $x^2 + y^2 = 1$. What is the set of all possible values of $2xy$?

All values between -1 and 1, including -1 and 1

5

An unfair coin is flipped twice. Let A denote the probability that you see one head, and one tail. Let B denote the probability that you saw either all heads or all tails. Which of the following are true: $A > B$, $A < B$, or $A = B$? $\boxed{B > A}$

Round 6 - Championship Round

1

Let n be the 100-digit integer, all of whose digits are 9. What is the remainder when n is divided by 7? $\boxed{\textbf{3}}$

2

Given 5 points: $A = (3, 5)$, $B = (0, -5)$, $C = (2, -8)$, $D = (10, 0)$, $E = (6, 5)$. Every minute all the points move by doubling their x-coordinates and halving their y-coordinates. They repeat this every minute for 1,000 years. At that time, which 2 points are closest? $\boxed{\textbf{A,C}}$

3

(use overhead) Simplify $\sqrt{4^{\frac{2}{\log_3 4}+2(\log_9 3)}}$. $\boxed{\textbf{6}}$

4

Suppose the sides of a regular tetrahedron have length 6. Find the distance between non-adjacent edges.

$\boxed{3\sqrt{2}}$

5

Let θ be the angle between any two faces of a regular tetrahedron. Find $\cos\theta$.

1/3

6

Six people – Jan, Jane, Gene, John, Joan, and June – decide to play a game, for which they must split up into two teams of three. In how many ways can they do this?

10

7

A famous mathematician was lying in the hospital when his friend, another mathematician, remarked that the number 1729 was not very interesting. The bed-ridden mathematician replied that on the contrary, 1729 is quite interesting: it is the smallest positive integer which can be written as the sum of the cubes of two positive integers in two different ways. What are these two different ways?

$\boxed{12^3 + 1^3, 10^3 + 9^3}$