

From Round 1**1** _____

L Find the area of a triangle with side lengths 3, 4 and 5 **6**

R Find the sum of the natural numbers from -50 to $+52$, including -50 and $+52$. **103**

2 _____

L Find the area of a trapezoid with sides of length 2, 3, 3 and 6, where the sides of length 3 and 6 are parallel. **$4\sqrt{5}$**

R I started with a prime $P > 30$. I squared P , then I squared that, then I squared that, and so on. Altogether, I squared 100 numbers. What was the last digit of the number I ended up with? **1**

3 _____

L What is the rightmost digit of 1997^{1997} ? **7**

R Find the area of a regular hexagon which has perimeter 12. **$6\sqrt{3}$**

4 _____

L What is the value of the infinitely repeating base-two number $0.01010101010101\dots$ **$1/3$**

R A $5 \times 5 \times 5$ cube is built out of $1 \times 1 \times 1$ cubes. If the outside surface of the large cube is painted, how many of the small cubes will have paint on exactly two faces?

36**5**

L If a block weighs 8 lbs plus half a block, how much does a block-and-a-half weigh?

24 lb

R At a party, there were 200 people, of whom 5% wore one earring, and of the other 95%, half wore no earrings and half wore two earrings. How many earrings were worn at this party?

200**6**

L A $5 \times 5 \times 5$ cube is built out of $1 \times 1 \times 1$ cubes. If the outside surface of the large cube is painted, how many of the small cubes will have paint on exactly one face?

54

R What is the only four-digit number in which the first digit is one third the second, the third is the sum of the first and the second, and the last is three times the second?

1,349**7**

L Find all positive solutions to $2^x = x^2$.

2,4

R Joe has two candy bars, and he offers to sell you either one candy bar for 40 cents or two for 70 cents. Either way, Joe makes the same total profit. If each candy bar cost him the same amount, how much is this amount?

30 cents

8

L (use overhead) What is the period of the graph of

$$y = 3 \tan\left(\frac{x}{3}\right) + 4?$$

 3π or 540 degrees**R** (use overhead) Express $(1 + i)^{11}$ in the form $a + bi$, where a and b are real and i is the square root of -1 . **$-32 + 32i$** **9**

L (use overhead) Compute the following expression.

$$\prod_{k=3}^9 \left(\frac{k}{k+1}\right).$$

 $3/10$ **R** (use overhead) What is the period of the graph of

$$y = 4 \cos(2x + 11\pi)?$$

 π or 180 degrees**From Round 2****1**

L A dog and cat together cost \$25. If the dog had cost \$5 more and the cat's price did not change, then the cat's price would have been a third of the total. How much did the dog cost?**\$15****R** What is the surface area of a cube with volume $27/8$? **$27/2$**

2

L I have a list of 5 positive integers whose median is 3 and whose mode is 4. Name the 5 numbers in my list.

1,2,3,4, and 4

R I took my cheetah out for a jog yesterday. He set the pace for the first mile, which we ran in 1 minute. I set the pace for the second mile. Our average speed for the 2 miles was 24 m.p.h. What was our speed for the second mile?

15 m.p.h.3

L A *great circle* is a circle drawn on a sphere whose center is also the center of the sphere. There are three great circles drawn on a sphere that don't all meet at any point. They divide the sphere into how many regions?

8

R (*use blackboard*) Find the area of the region in the plane satisfying both $|x - y| \leq 1$ and $|x + y| \leq 1$.

24

L Find the smallest possible area of triangle ABC if $AB = 12$, $BC = 10$ and $\angle BAC = 30^\circ$.

 $18\sqrt{3} - 24$

R Given an isosceles right triangle with hypotenuse of length $\sqrt{2}$. What is the distance between the center of the inscribed circle and the center of the circumscribed circle?

 $1/(2 + \sqrt{2}) = (2 - \sqrt{2})/2$ 5

L How many positive integers less than 600 are not a

multiple of 2, 3, or 5?

160

R Let a denote $\log_2 10$ and b denote $\log_3 10$. In terms of a and b , what is $\log_{10} 150$?

$2 + (1/b) - (1/a)$

6

L (*use overhead*) Simplify the following expression, where i is the square root of -1 .

$i/2$

$$\left(\frac{1}{i} - i^{-3}\right)^{-1},$$

R (*use overhead*) The following picture is not drawn to scale. If the measures of angles A , B , C , D , E and F are 10, 20, 30, 30, 20 and 15 degrees, respectively, what is the measure of angle G ?

55 degrees

7

L (*use overhead*) A square is inscribed in a circle which is inscribed in a square, as seen in the diagram. What is the ratio of the areas of the small square to the large square?

1:2

R (*use overhead*) How many degrees are in an angle of a regular polygon with 15 sides?

156

8

L (*use overhead*) For how many y does the following

1

equation have an odd number of real solutions for x ?

$$(x^2 - 1)^2 = y$$

R (use overhead) Find

1/2

$$f(f(f(f(\dots f(10)\dots))))),$$

where the above expression contains 97 f s and $f(x) = \frac{5}{x}$.

From Round 3

1

There is a triangle in three-space. Its vertices have coordinates $(0,0,0)$, $(1,3,4)$, and $(-1,3,4)$. Find the area of the triangle.

5

2

The cost of a widgit plus its box is \$1.10, and the widgit costs \$1.00 more than its box. How much does the box cost?

\$0.05

3

Find the distance between the points $(1, 2, 3)$ and $(3, 4, 5)$ in three-dimensional space.

$\sqrt{12} = 2\sqrt{3}$

4

Four positive numbers are in geometric progression. If the

$\boxed{4/3}$

product of the first and third numbers equals 4 and the product of the second and fourth numbers equals 9, then what is the value of the first number?

5

(use overhead) You fold a 1-inch square piece of paper along its diagonal, then in half and then in half again. What are the lengths of the sides of this new shape?

 $\boxed{1/2, 1/2, \sqrt{2}/2}$ **From Round 4**

1

Today is Saturday, April 26, 1997. The year 2000 is a leap year. What day of the week will April 26, 2001 fall on?

 $\boxed{\text{Thursday}}$

2

A 10-foot ladder is leaning against a wall, with its base 5 feet from the wall. If the bottom of the ladder slips 1 more foot away from the wall, then how far will the top of the ladder move?

 $\boxed{5\sqrt{3} - 8}$

3

What is the probability of tossing 3 standard dice and getting a sum of 15?

 $\boxed{10/6^3 = 10/216 = 5/108}$

4

A woman and her two children need to cross a river. They have a rowboat, but this boat can only carry a maximum of 150 lbs. If the mother weighs 130 lbs and each child weighs 75 lbs, what is the least number of river crossings required to get the entire family across the river?

5

5

The number x written in base 7 is $.33333333\dots$. What is x in base 5?

.222222...

6

Find $\log_3(4^{\log_2 3})$. (“Log base 3 of the quantity 4 to the log base 2 of 3.”)

2

7

Randomly choose a point on the unit-circle, and draw a line through it and the origin. What is the probability that the line has slope less than $\sqrt{3}/3$?

2/3

From Round 5

1

A ball is removed at random from an urn which has 10 white and 10 black balls, and is not replaced in the urn. This process is repeated twice. What is the probability that both balls are white?

9/38

2

P is a regular 1997-gon with side-length 1. Let S be the region in the plane consisting of all points outside of P whose distance from P is less than 1. Find the area of S.

$1997 + \pi$

3

Find the maximum value of the function

13

$$f(x) = 5 \sin x + 12 \cos x,$$

where x ranges over all real numbers.

4

Three people — A, B and C — sit in a circle and successively toss a fair coin in the order A, B, C, A, B, C, etc. The winner is the first person to throw a “head.” What is the probability that person A wins this game?

$4/7$

5

What kind of curve is the polar coordinates graph of $r = \cos \theta + \sin \theta$?

a circle

6

Find to the nearest hundredth the cubed root of 8,024.

20.02

From Championship Round

1

A and B each throw a die. What is the probability that A's

5/12

die shows a greater value than B's die?

2 _____

Find seven non-zero integers whose sum and product are equal.

eg: -1,1,-1,1,1,2,3**3** _____

It takes me 10 minutes to eat a big mac. However, if I ever consume any amount over 2 big macs within an hour, I will get sick. What is the fastest I can eat 6 big macs without getting sick?

140 minutes**4** _____

An ice sculpture of a puppy weighs 2000 pounds and stands 5'10" tall. Another ice sculpture has exactly the same proportions as the first, but is 6'5" tall. How much does it weigh?

2662 pounds**5** _____

Six people — Bob, Bobbie, Rob, Robbie, Robert and Roberta — are to be divided into two study groups. The groups cannot have any person in common, and each group must contain at least one person. In how many ways can this be done?

31**6** _____

A ball is removed at random from an urn which has 10 white and 10 black balls, and *not* replaced in the urn. This process is repeated 4 times. What is the probability that the third ball was white?

1/2

7

I have a standard 12-hour wall clock with an hour hand and a minute hand. The clock runs slow: in one hour of actual time my clock shows that 58 minutes have elapsed. If my clock is showing the correct time on Monday, Mar. 1 at 6 A.M., when will it next show the correct time?

Mar 16 at 6 A.M.