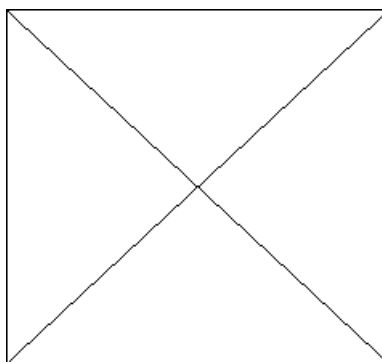


# Test of Ingenuity



BAY AREA MATH MEET  
UNIVERSITY OF SAN FRANCISCO  
April 29, 2000

**Directions:** Please do not open the test booklet until the proctor begins the examination. This is a multiple-choice, 20-question exam. You will have one hour to work on the problems. You get one point for a correct answer, zero points for no answer, and  $-1/4$  points for an incorrect answer. Because of this penalty for guessing, if you are not sure of the correct answer to a question, it is best not to answer the question.

The questions are arranged in roughly increasing order of difficulty. The last 6 questions are particularly hard and will be used to break ties. Unless you are extremely ambitious, do not attempt all of the problems! Answering just half of them correctly is a very fine achievement! Read the questions first to see which problems are best for you.

Good luck, and have fun!

1 Find the area of a circle with circumference  $12\pi$ .

A  $6\pi$

B  $12\pi$

C  $24\pi$

D  $36\pi$

E  $144\pi$

2 Compute  $1 + 2 + 3 + \cdots + 1998 + 1999 + 2000$ .

A 2,000,000

B 2,001,000

C 2,002,000

D 4,000,000

E 4,002,000

3 Compute the following product. The values, including your answer, are written in base-5.

$$23 \times 32 =$$

A 1231

B 1241

C 1341

D 1441

E 2001

4 Let  $f(x) = x^2 + 1$ . Then  $f(y^2 + 1) =$

A  $y^4 + 1$        B  $y^4 + y^2 + 1$        C  $y^4 + 2y^2 + 1$

D  $y^4 + 2y^2 + 2$        E  $y^4 + 4y^2 + 4$

5 Two runners, A and B, go around a 1-mile track in same direction. During one hour, B passed A three times. If each person runs at a constant speed, then which of the following statements must be true?

A B is exactly 3 times as fast as A

B B is 3 mph faster than A

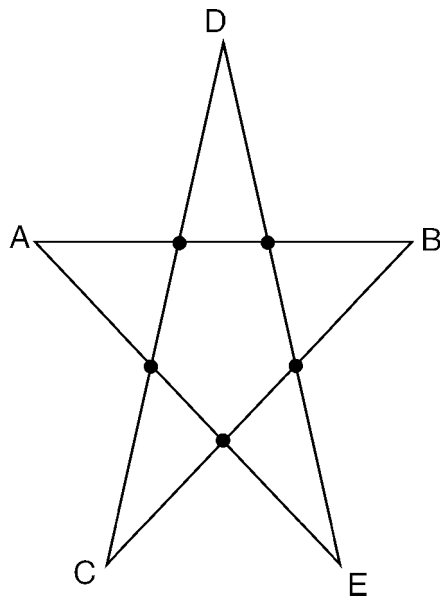
C B is less than 3 mph faster than A

D B is more than 2 mph faster, and less than 4 mph faster than A

E B is more than 3 mph faster, and less than 4 mph faster than A

- 6 Line segments  $AB$ ,  $BC$ ,  $CD$ ,  $DE$ , and  $EA$  form a 5-pointed star as shown below (note: the figure is not drawn to scale). Suppose that the measures of the interior angles of the pentagon (vertices marked with dots) are in arithmetic progression. Find the sum of the measures of angles  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$  (in degrees).

- A 240       B 210       C 180       D 165
- E Cannot be uniquely determined from the given information.



7 A sphere is cut by parallel planes into  $n$  pieces. It turns out that the total surface area of these pieces is exactly 2000 times the area of the sphere. What is the smallest possible value for  $n$ ?

A 1999

B 2000

C 3141

D 3999

E 4000

8 Amanda deposits \$1000 at the end of each year into a bank account. The account pays 10% interest at the end of each year on funds which have been in the account during the past year. How much is in her account after she makes her fourth payment? (Assume that the account was empty before she started her payments.)

A \$4641

B \$4687

C \$4331

D \$4651

E \$4421

9 In the nation of Klopstockia, 90% of the adults drink tea, 80% drink coffee, 70% drink wine, and 60% drink beer. No one drinks all four beverages. Let  $p$  be the percent of Klopstockian adults that drink either beer or wine (or both). Which of the following statements must be true?

A  $p = 100$        B  $p = 80$        C  $80 < p < 90$

D  $90 < p < 100$        E  $p = 60$

10 How many zeros are at the end of  $2000! = 1 \cdot 2 \cdot 3 \cdots 1999 \cdot 2000$ ?

A 400       B 449       C 480       D 499       E 512

**11** Let  $P(x)$  be a polynomial of degree 2000, such that  $P(1) = P(2002) = 1$  and

$$P(2) = P(3) = P(4) = \cdots = P(2000) = P(2001) = 0.$$

Find  $P(0)$ .

A 2000       B 2001       C  $1/2000!$

D  $-1/2000!$        E  $2001!$

**12** Five people take a 10-question exam. For each question, exactly 3 people gave the correct answer, and 2 gave the incorrect answer. What are the possible values for the median exam score? (Each person's exam score is the number of questions that he or she answered correctly.)

A Only 6       B Only 5 or 6       C Only 5, 6, or 7

D Only 5, 6, 7, 8, 9, or 10       E Only 4, 5, 6, 7, 8, 9, or 10

**13** The graph of  $|3x^2 + y^2 - 12| = |x^2 - y^2 + 4|$  is

- A** a union of a circle and two lines.
- B** a union of an ellipse and a hyperbola.
- C** a circle.
- D** a union of two circular arcs.
- E** a union of a circle and a line.

**14** Let  $x$ ,  $y$ , and  $z$  be positive real numbers such that

$$\log_{2x} z = 3, \quad \log_{5y} z = 6, \quad \log_{xy} z = 2/3.$$

Find  $z$ .

- A**  $\frac{1}{\sqrt[3]{5}}$
- B**  $\frac{1}{10}$
- C**  $\frac{2}{\sqrt[6]{5}}$
- D**  $\frac{1}{2}$
- E**  $\frac{1}{5}$

**15** Let  $f(x)$  be a function with domain  $0 \leq x \leq 1$  defined by

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1/2, \\ -2x + 2, & \text{if } 1/2 < x \leq 1. \end{cases}$$

Find the area of the set of all points  $(x, y)$  such that

$$0 \leq x \leq 1 \quad \text{and} \quad 0 \leq y \leq f(f(f(x))).$$

**A** 1

**B**  $1/2$

**C**  $1/4$

**D**  $4/9$

**E**  $1/8$

**16** Let  $x = \sqrt[3]{1000} - \sqrt[3]{999}$ . What integer is closest to  $1/x$ ?

**A** 200

**B** 250

**C** 300

**D** 333

**E** 400

**17** Let  $P$  be a regular 20-gon which is inscribed in a circle of radius 1. Find the sum of the squares of the lengths of all of the sides and diagonals of  $P$ .

A 400       B  $2048 \sin 9^\circ$        C  $625 \sin 18^\circ$

D  $32 \csc 9^\circ$        E  $128(1 + \sqrt{2})$

**18** Let  $N = 10^{2000} + 2000$ , and let  $S$  be the set of consecutive integers from 1 to  $N$ . In how many different ways can three consecutive integers be removed from  $S$  so that the average of the remaining numbers is an integer?

A 1       B 2       C 3       D 4       E 6

**19** An urn contains 17 balls, labeled 1, 2, 3, ..., 17. A person shakes the urn, selects a ball, records the number, and then replaces the ball. This is done 17 times. The average (expected) number of *different* values recorded is less than 17 (for example, it is possible that the same number will be selected 17 times in a row, in which case just one value is recorded). If this expected value is written as a fraction in lowest terms, find the last (rightmost) digit of the sum of the numerator and denominator.

A 1

B 2

C 3

D 5

E 8

20 Consider the following experiment:

- First a random number  $p$  between 0 and 1 is chosen by spinning an arrow around a dial which is marked from 0 to 1. (This way, the random number is “uniformly distributed”—the chance that  $p$  lies in the interval, say, from 0.45 to 0.46 is exactly  $1/100$ ; and the chance that  $p$  lies in the interval from 0.324 to 0.335 is exactly  $11/1000$ , etc.)
- Then an unfair coin is built so that it lands “heads up” with probability  $p$ .
- This coin is then flipped 2000 times, and the number of heads seen is recorded.

What is the probability that exactly 1000 heads were recorded?

A  $\frac{2000!}{(1000!)^2 2^{1000}}$

B  $\frac{1000!}{(500!)^2 2^{500}}$

C  $\frac{1}{2^{1000}}$

D  $\frac{1}{1001}$

E  $\frac{1}{2001}$

## Answers

1.  $36\pi$
2. 2,001,000
3. 1341
4.  $y^4 + 2y^2 + 2$
5. B is more than 2 mph faster, and less than 4 mph faster than A
6. 180
7. 4000
8. \$4641
9.  $p = 100$
10. 499
11. 2001
12. 4–10, inclusive
13. a union of a circle and two lines.
14.  $1/10$
15.  $1/2$
16. 300
17. 400
18. 4
19. 2
20.  $1/2001$