ANSWER KEY TO TEST OF INGENUITY
BAMM, 30 APRIL 1994

1. **B** 2. **C**
3. **B**. Look at the problem from Jane's point of view. Pretend that she is standing still. Then Mary is traveling (with respect to Jane) at a speed of 60/2 - 60/2.25 = 3.33 laps per hour...
4. **D**
5. **D**. Don't forget that divisible by 7 or 11 means you have to count the number divisible by 7 plus the number divisible by 11 *minus* the number divisible by both (77). Why?
6. **C.** 100-95 = 5 people who don't speak Russian. Likewise, 20 don't speak Mandarin, and 10 don't speak Spanish. Some of these may overlap, but if there is no overlap, then it is possible for at most 5 + 20 + 10 =35 people to not speak all three languages. So at least 100-65 do speak all three languages.
7. **C**. Make sure that you rotate the squares so that it becomes immediately obvious that the ratio of the areas is 2:1!
8. **B**. It is good to be familiar with the easy-to-verify formula for the area of an equilateral triangle: $A = \sqrt{3}\, s^2/4$, where s is the length of a side.
9. **C**
10. **E.** The number of zeros is the highest power of 10 that divides 150!. Since 10 = 5 x 2, and there are plenty of 2's available, the "bottleneck" is the highest power of 5 which divides 1 x 2 x 3 x ... x 150. First, divide 150 by 5 to get 30 multiples of 5. But that is not enough, because we need to count the multiples of 25 *twice.* There are 6 of these. We are almost done, but we also need to count the single multiple of 125. It was counted once when we counted the 30 multiples of 5, counted once more when we counted the six multiples of 25, but needs to be counted one more time. So the total is 37.
11. **C.** Rationalize the numerator!
12. **D.** Call the sum S. Figure out 2S, then subtract. You will have something that you should recognize.
13. **D**
14. **E.** Use the fact that c = -(a+b). Then $a^3 + b^3 + c^3 = -3ab(a+b)$. Once again, use c = -(a+b).
15. **E.** Use the "reflection trick": Draw a path, reflect it with respect to the x- and y-axes, and use the fact that the shortest distance between two points is a straight line.
16. **B.** From the bug's point of view, after time 0 it walks 1/3 of the band's length. After time 1minute, it walks 1/4 of the band's length, after time 2 minute, it walks 1/5 of the band's length, etc. So we want to know when the sum 1/3 + 1/4 + 1/5 + 1/6 + ... will first exceed 1.
17. **A**. This is not as hard as it looks. Note that the stuff about Lisa is just a "red herring." Let j, m, b equal the ages of John, Mary, Bill today. How many years ago was Bill "three times as old as Lisa was when she was 17 years younger than John is now"? That's easy -- it is b - 3(j-17). Why? Keep going, and you should get the simultaneous equations j = 2(m-b+ 3(j-17)) and m-b = 6.
18. **C.** Try writing a_{100} in terms of a_{99}, and then keep applying the recursion formula, to get a_{100} in terms of a_{99}, a_{98}, etc. You will get a geometric series ...
19. **A.** Amazingly, it doesn't even depend on how well you stir the liquids, just as long as the amounts are the same. The easiest way to see this is to imagine a discrete problem. Let container A have 100 white ping-pong balls and let container B have 100 black ping-pong balls. If you move, say 5 balls from A to B and then return *any* 5 back from B to A, it is easy to see that the number of black balls in A must equal the number of white balls in B!
20. **A.** This is a hard problem, and we will only give a sketch here. Trace the "orbit" of the top card, which starts in position 1. It goes to position 2, then 4, etc. Check that after r shuffles, it goes to position 2^r (mod 95). (*Not* mod 94!) The minimum number of shuffles will be the minimum r such that 2^r is congruent to 1 (mod 95). By Euler's theorem, r must divide $\phi(95) = 72$. It turns out that 36 is the smallest such exponent.