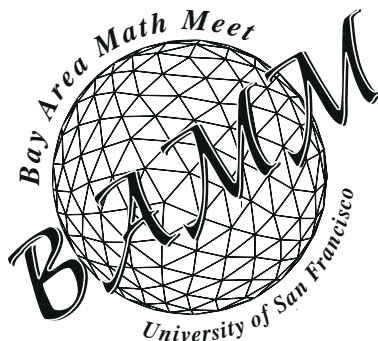


Test of Ingenuity



BAY AREA MATH MEET
UNIVERSITY OF SAN FRANCISCO
April 25, 1998

Directions: Please do not open the test booklet until the proctor begins the examination. This is a multiple-choice, 20-question exam. You will have one hour to work on the problems. You get one point for a correct answer, zero points for no answer, and $-1/4$ points for an incorrect answer. Because of this penalty for guessing, if you are not sure of the correct answer to a question, it is best not to answer the question.

The questions are arranged in roughly increasing order of difficulty. The last 6 questions are particularly hard and in the event of a tie score, the first prize will go to the student with the highest score on these six problems. Unless you are extremely ambitious, do not attempt all of the problems! Answering just half of them correctly is a very fine achievement! Read the questions first to see which problems are best for you.

Good luck, and have fun!

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1 What is the area of a circle with circumference 8π ?

- A 16 B 8π C 16π D 32π E 64π

2 How many 4-digit base-10 numbers end with the digits "98"?

- A 72 B 81 C 90 D 99 E 100

3 Let $f(x, y) = x^2 - y^2$. Then $f(x + y, x - y) =$

- A 0 B $4y^2$ C $2xy$ D $4xy$ E $x^2 + y^2$

4 The lengths of a triangle are 5, 12, and 13. Find the length of the altitude to the longest side.

A 3

B 4

C 5

D $60/13$

E $\sqrt{74}$

5 Today is Saturday, April 25, 1998. What is the next year in which April 25 will also be a Saturday? Don't forget that the years 2000, 2004, 2008, etc. are leap years.

A 2004

B 2005

C 2007

D 2009

E 2010

6 Eric and Samuel were born on the same day of the month, but on different months. Eight months ago, Eric was 4 times as old as Samuel, and in two months, Samuel will be twice as old as Eric was when Samuel was born. What is the sum of their ages, in months?

A 12

B 14

C 24

D 26

E 36

- 7 A haunted house has only one entrance door. This door opens into a room which has two more doors, and it turns out that every room in this house has exactly three doors. The house has n rooms. Which of the numbers below are possible values for n ?

A 64 B 28 C 27 D 32 E All of these.

- 8 Lorenzo drives one mile down the road from his house, traveling at 30 miles per hour. If he wishes to have an average speed of 60 miles per hour for the round trip, how fast should he travel on the return trip?

A 15 mph B 30 mph C 60 mph D 90 mph
 E It is impossible no matter how fast he drives.

- 9 Compute the value of the infinite series

$$\frac{1}{1^2 + 1} + \frac{1}{2^2 + 2} + \frac{1}{3^2 + 3} + \frac{1}{4^2 + 4} + \dots$$

A $\frac{1}{2}$ B 1 C $\frac{\pi^2}{6}$ D $\frac{\pi^2}{8}$ E $1 + \frac{\pi^2}{6}$

- 10** Chickenoid Nuggitz from Bay Area Rapid Food can be ordered in buckets of 6, 9, or 20 chicken chunks. What is the largest number n such that no combination of Chickenoid Nuggitz will yield exactly that number of chicken chunks? For example, it is possible to get 12 chicken chunks, but it is not possible to get 10 chunks.

A 38 B 41 C 43 D 44 E 53

- 11** A robot is programmed to shuffle the cards of a 10-card deck. Unfortunately, this robot always performs exactly the same shuffle each time. It turns out that it takes n shuffles for the deck of cards to be restored to its original order for the first time. Of the numbers below, which one cannot be a possible value for n ?

A 7 B 14 C 18 D 21 E 30

- 12** Let R be the region of points (x, y) in the plane which satisfy the inequality

$$|x| - 1 + |y| - 1 \leq 1.$$

What is the area of R ?

A 1 B 2 C 4 D 8 E 16

13 A shed consists of a 10-foot cube with a pyramid roof. When viewed from a great distance, directly opposite the center of a side, all that one sees is an equilateral triangle on top of a square. What is the surface area of the roof of the shed (in square feet)?

A 100 B $100\sqrt{2}$ C $100\sqrt{3}$

D 200 E $100\left(\frac{\sqrt{2} + \sqrt{3}}{2}\right)$

14 A virus is placed into a colony of 1,998 bacteria. Every minute, each virus destroys one bacterium apiece, after which all the bacteria and viruses divide in two. For example, after one minute, there will be $1997 \times 2 = 3994$ bacteria and 2 viruses. After two minutes, there will be 3992×2 bacteria and 4 viruses, etc. How long (in minutes) will it take for the all the bacteria to be destroyed?

A 999 B 1,997 C 1,998 D 1,999 E never

15 A random number generator outputs the values 1, 2, 3, 4, 5, with each value equally likely. If three numbers are output, what is the probability that their product is a multiple of 6?

A $\frac{27}{125}$

B $\frac{32}{125}$

C $\frac{36}{125}$

D $\frac{42}{125}$

E $\frac{48}{125}$

16 A bag contains two red balls, two green balls, two white balls, two black balls, and two brown balls. Other than color, the balls are indistinguishable. In how many different ways can one select 5 balls from this bag?

A 48

B 51

C 53

D 60

E 252

17 Each of the following numbers is a prime, and all but one of them can be expressed as the difference of two cubes of positive integers. Which one?

A 25,366,109

B 2,465,227

C 58,887,991

D 25,675,651

E 1,114,471

18 For n a positive integer, let $\sigma(n)$ be the sum of the positive integer divisors of n including 1 and n . For how many positive integers n between 2 and 300, inclusive, is $\sigma(n)$ odd?

A 16

B 24

C 28

D 30

E 32

- 19 The set of points (x, y) which satisfies $x^3 - 3xy^2 \geq 3x^2y - y^3$ and $x + y = -1$ is a line segment. Find its length.

- A $\sqrt{6}/4$ B $2\sqrt{2}/3$ C $3(\sqrt{3} - \sqrt{2})$ D $\sqrt{6}/3$
 E 1

- 20 In the kingdom of Klopstockia, every day is either completely sunny or completely rainy, and each day, the probability is $2/3$ that the weather will be the same as it was the previous day. Find the probability that April 5 was sunny, given that both April 1 and April 9 were sunny.

- A $\frac{1}{2}$ B $\frac{25}{41}$ C $\frac{625}{1681}$ D $\frac{576}{1301}$ E $\frac{1681}{3281}$